

How to complete the square

[The objective is to change the equation from quadratic form to completed the square form]

The completed the square form looks like this $\pm a(x \pm b)^2 \pm c$

Examples:

- 1) $(x - 3)^2 + 4$
- 2) $2(x + 4)^2 - 6$
- 3) $-\left(x - \frac{3}{4}\right)^2 - 3\frac{5}{7}$

Example 1

$$x^2 + 3x + 6 = 0 \text{ [original equation]}$$

- Always ensure that the quadratic equation is **equal to 0** before completing the square
- Always ensure that the coefficient of x^2 is **1**
- The coefficient of x is **3**.

[Recall that a Quadratic Equation comes in this form $ax^2 + bx + c$]

- Hence in this example, $b = 3$ and $c = 6$

$$\left(x \pm \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 \pm c = 0 \text{ [General Formula]}$$

In this example:

$$b = 3 \text{ and } c = 6$$

$$\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 6 = 0$$

- 1) The first sign (yellow highlight) **always follows the sign** in the original equation
- 2) The second sign (blue highlight) is **always a negative sign**
- 3) The third sign (green highlight) **always follows the sign** in the original equation

$$\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 6 = 0$$

$$\left(x + \frac{3}{2}\right)^2 - 3\frac{3}{4} = 0$$

[Completed the square form]

Example 2

$$x^2 - 4x - 6 = 0 \quad \text{Quadratic Equation}$$

$$\left(x - \frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 6 = 0$$

$$(x - 2)^2 - 4 - 6 = 0$$

$$(x - 2)^2 - 10 = 0 \quad \text{Completed the square form}$$